When θ is large the velocity in the direction of the force becomes

$$U_h = \frac{Xe}{m} \cdot \frac{1}{\omega^2 T} = \frac{Xm}{H^2 e T}, \quad \text{so that} \quad \tan\theta = \frac{HeT}{m} = \frac{X}{HU_h}.$$

In this case the arc of the circle described between two collisions is large compared with the radius.

The velocities of ions U under an electric force in gases at low pressures may thus be easily determined by producing a small deflection θ of a stream with a magnetic force, since $U = U_h$ when θ is small. Also the theory may be tested by observing the effect of a magnetic force on the diffusion of a narrow stream moving under an electric force. In this case when the magnetic force coincides with the electric force, the motion arising from diffusion in directions normal to the force is reduced in the proportion K_h/K, so that the ions are kept together in a narrower stream.

There are several well known phenomena connected with the magnetic rays that occur in discharge tubes, in which remarkable effects are obtained by magnetic forces.

The observed effects are qualitatively in accordance with the above theory, but the conditions under which they are obtained and the system of forces that is called into play are so complicated that an accurate comparison with the theory would not be possible.

A New Treatment of Optical Aberration.

By R. A. Sampson, F.R.S.

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(Abstract.)

A method is developed by which Gauss's method of relating original and emergent rays in a coaxial optical system

$$\left.\begin{aligned} y &= \beta x + b, \\ z &= \gamma x + c, \end{aligned}\right\} \qquad \left.\begin{aligned} y' &= \beta' x' + b', \\ z' &= \gamma' x' + c', \end{aligned}\right\}$$

by means of a transformation,

$$b' = Gb + H\beta, \qquad \beta' = Kb + L\beta, \qquad c' = Gc + H\gamma, \qquad \gamma' = Kc + L\gamma,$$

where $$GL - HK = \mu/\mu' = N,$$

may be applied so as to include the aberrations of the third order.

Using $\delta G, \ldots$, for the added terms for a single surface, of curvature B, with both origins at the tangent plane,

$$G+\delta G = 1+\omega, \quad H+\delta H = \omega/B, \quad K+\delta K = -(1-N)B+B(\psi-\epsilon\omega),$$
$$L+\delta L = N+\psi-\omega,$$

where $\quad \omega = \tfrac{1}{2}(1-N)B^2(b^2+c^2), \qquad \psi = \tfrac{1}{2}N(\beta'^2+\gamma'^2-\beta^2-\gamma^2),$

and ϵ is a term which is unity for the sphere and zero for the paraboloid.

It is shown how to build up the values resulting from any series of such aberrations and to obtain finally the aberrational increments $\delta G, \ldots$, in the forms

$$\delta G = \tfrac{1}{2}\{\delta_1 G(b^2+c^2)+2\delta_2 G(b\beta+c\gamma)+\delta_3 G(\beta^2+\gamma^2)\},$$
$$\delta H = \text{etc.}, \qquad \delta K = \text{etc.}, \qquad \delta L = \text{etc.}$$

The twelve coefficients, $\delta_1 G, \ldots$, are then shown to obey seven relations, namely,

$$\frac{\delta_2 G-\delta_1 H}{G} = \frac{\delta_3 G-\delta_2 H}{H} = \frac{\delta_2 K-\delta_1 L}{K} = \frac{\delta_3 K-\delta_2 L}{L} = \mathfrak{P},$$

where $\quad \mathfrak{P} = \mu_{-1}\Sigma(\mu_{2r-1}{}^{-1}-\mu_{2r-1}{}^{-1})B_{2r},$

and is in fact Petzval's expression, the vanishing of which is known as the condition for flatness of field in stigmatic systems, and

$$\begin{aligned}\delta_1 N &= G\delta_1 L+L\delta_1 G-H\delta_1 K-K\delta_1 L = K^2N,\\ \delta_2 N &= \ldots \qquad\qquad\qquad\qquad\qquad\quad = KLN,\\ \delta_3 N &= \ldots \qquad\qquad\qquad\qquad\qquad\quad = (L^2-1)N.\end{aligned}$$

These lead to Abbe's sine condition, and both throw light on the general relationships of these two well known conditions.

The geometrical interpretation of the presence of the coefficients $\delta_1 G, \ldots$, at any numerical values, is followed out. The formulæ are then applied, as a numerical illustration, to the calculation of the celebrated Fraunhofer heliometer objective described by Bessel, and calculated with great completeness by A. Steinheil.* In this portion the whole numerical work is given, and it is shown to amount to a mere fraction of that requisite for the trigonometrical calculation as employed, *e.g.*, by Steinheil. Comparison is made with the whole of Steinheil's results, for rays which meet and which do not meet the axis; the numbers are shown to agree within a few units in the last places, and this discrepancy is probably to be attributed to the large number of operations requisite for the trigonometrical calculation. Thus the method is adequate for the numerical calculation of telescopic objectives, and offers a remarkable economy in the work hitherto necessary.

* 'Munich. Akad. Math. Phys. Class. Sitzungsber.,' vol. 19, Part 3.